

Behaviour of the ΛN - and ΛNN - potential strengths in the ${}^5_\Lambda\text{He}$ hypernucleus

A. A. Usmani*

*Department of Physics, Aligarh Muslim University, Aligarh 202 002, India and
Inter University Centre for Astronomy and Astrophysics (IUCAA), Ganeshkhind, Pune-411 007, India*

F. C. Khanna†

*Physics Department, Theoretical Physics Institute,
University of Alberta, Edmonton, T6G 2J1, Canada and
TRIUMF, 4004, Wesbrooke Mall, Vancouver, British Columbia, V6T 2A3, Canada
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Variational study of the ${}^5_\Lambda\text{He}$ hypernucleus is presented using a realistic Hamiltonian and a fully correlated wave function including ΛN space-exchange correlations. Behaviour of Λ -separation energy (B_Λ) with two- and three- baryon potential strengths is thoroughly investigated. Solutions for these potential strengths giving experimental B_Λ are presented.

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Recently, a variational study [1] of the ${}^5_\Lambda\text{He}$ hypernucleus has been performed with a realistic Hamiltonian and a fully correlated wave function (WF). The WF takes into account all relevant dynamical correlations induced by the two- and three- baryon potentials and the ΛN space-exchange correlation (SEC) that arises due to the space-exchange potential. The findings of the investigation suggest that no realistic study ignoring SEC is fair as it significantly affects every physical observable like energy breakdown, Λ -separation energy (B_Λ), nuclear core polarization (NCP), point proton radius and density profiles. The effect is found more evident in the ${}^6_{\Lambda\Lambda}\text{He}$ double- Λ hypernucleus [2]. The ground-state energy of the hypernucleus (E) or the Λ -separation energy ($B_\Lambda = E_{\text{He}} - E$) depends on the strengths of the potentials involved in the Hamiltonian.

A realistic Hamiltonian H of the hypernucleus, is written as a sum of the Hamiltonians due to the nuclear core (NC) of the hypernucleus (H_{NC}) and due to the Λ -baryon (H_Λ),

$$H = H_{NC} + H_\Lambda, \quad (1)$$

$$H_{NC} = T_{NC} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk}, \quad (2)$$

$$H_\Lambda = T_\Lambda + \sum_i v_{\Lambda i} + \sum_{i < j} V_{\Lambda ij}. \quad (3)$$

Here, subscripts i, j and k refer to nucleons. For the $S = 0$ sector, we use Argonne v_{18} NN potential [3] and Urbana type NNN potential [4, 5], which successfully explain the nuclear energy spectra and are well established. However, for the $S = -1$ sector, ΛN and ΛNN potential strengths are yet to be determined. The dependence of energy on these strengths is the theme of this work.

Variation in any of the potential strengths would directly affect the expectation value of the respective potential. It would also affect the WF as correlations are solutions of these potentials. Moreover, there are sensitivities to operators among various terms of the Hamiltonian and the correlation functions. Correlations like SEC bring-in changes in the density profiles which affect even the central pieces of the energy breakdown. The basic ingredients like these strengths, therefore affect the ground-state energy collectively. It would not be possible to perform a proper study for a particular potential strength ignoring others. Thus, in order to pin down these potential strengths, we have to handle them all together. The energy of ${}^5_\Lambda\text{He}$ is thoroughly investigated along these lines. Such a study of all the s -shell single- and double- Λ hypernuclei may help to pin down these strengths, which, in turn, may resolve the outstanding $A = 5$ anomaly [6, 7, 8] with no additional effort. Thus the present investigation is a step forward to answer, (i) whether we can successfully reproduce the hypernuclear energy spectra using these potentials without invoking the underlying Quantum Chromodynamics? (ii) and whether there is a possibility of physical existence of a bound ($I = 0, J = 1^+$) ${}^4_{\Lambda\Lambda}\text{H}$ hypernucleus which has been recently conjectured [9]? This would be helpful in studying the heavy hypernuclei specially ${}^{209}_\Lambda\text{Pb}$ whose core density matches the nuclear matter density. Hence, this would lead us to investigate, in detail, the physics of charmed and bottom hypernuclei [10, 11, 12] as well as to the nature and structure of neutron stars.

The ΛNN potential arises from projecting out Σ, Δ , etc., degrees of freedom from a coupled channel formalism. This is written as a sum of two terms, $V_{\Lambda ij} = V_{\Lambda ij}^D + V_{\Lambda ij}^{2\pi}$, as in Fig. 1. The dispersive potential $V_{\Lambda ij}^D$, arising from the suppression mechanism owing to $\Lambda N - \Sigma N$ coupling [13, 14, 15, 16], is written including the explicit spin dependence as [17]

$$V_{\Lambda ij}^D = W^D T_\pi^2(m_\pi r_{\Lambda i}) T_\pi^2(m_\pi r_{\Lambda j}) [1 + \sigma_\Lambda \cdot (\sigma_i + \sigma_j)/6]. \quad (4)$$

*Electronic address: anisul@iucaa.ernet.in

†Electronic address: khanna@phys.ualberta.ca

However, spin term is too weak for spin zero core nucleus. Here, W^D is the strength. The $V_{\Lambda ij}^{2\pi}$ is a two-pion exchange attractive potential. Neglecting higher partial waves it is written as a sum of two terms representing p- and s-wave $\pi - N$ scatterings, $V_{\Lambda ij}^{2\pi} = V_{\Lambda ij}^P + V_{\Lambda ij}^S$ as in Ref. [18]. The explicit form of these potentials are

$$V_{\Lambda ij}^P = -(C^P/6)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\{X_{i\Lambda}, X_{\Lambda j}\}, \quad (5)$$

and

$$\begin{aligned} V_{\Lambda ij}^S &= C^S Z(m_\pi r_{i\Lambda}) Z(m_\pi r_{j\Lambda}) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{i\Lambda} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{j\Lambda} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ &\equiv C^S O_{\Lambda ij}^S \end{aligned} \quad (6)$$

with

$$X_{\Lambda i} = (\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i) Y_\pi(m_\pi r_{\Lambda i}) + S_{\Lambda i} T_\pi(m_\pi r_{\Lambda i}) \quad (7)$$

and

$$Z(x) = \frac{x}{3} [Y_\pi(x) - T_\pi(x)]. \quad (8)$$

It may be expressed as generalised tensor-tau type operators $(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{\Lambda i})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{\Lambda j})$, $(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{\Lambda i})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{\Lambda i})$, $(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{\Lambda j})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{\Lambda j})$, and $(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$ followed by $(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)$, thus has a strong tensor dependence. In the above expressions, $S_{\Lambda i}$ is the tensor operator, $Y_\pi(x)$ is the Yukawa function

$$Y_\pi(x) = \frac{e^{-x}}{x} \xi_Y(r), \quad (9)$$

and $T_\pi(x)$ is the one-pion exchange tensor potential

$$T_\pi(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right) \frac{e^{-x}}{x} \xi_T(r). \quad (10)$$

Here, $\xi_Y(r)$ and $\xi_T(r)$ are short-range cut-off functions,

$$\xi_Y(r) = \xi_T^{1/2}(r) = (1 - e^{-cr^2}). \quad (11)$$

Here, $c = 2.0 \text{ fm}^{-2}$ is a cut-off parameter and subscripts i, j and Λ refer to two nucleons and a Λ in the triplet (Λij) . The C^P and the C^S are the strengths of $V_{\Lambda ij}^P$ and $V_{\Lambda ij}^S$, respectively. The latter is a very weak term compared to the former. Its strength is not known experimentally. However, we may make a qualitative theoretical estimate for it by comparing the strengths of modern NNN potentials obtained using $SU(3)$ symmetry. Using chiral perturbation theory, Friar *et al.* [19] have compared the modern NNN potentials, namely: (i) Tucson-Melbourn [20], (ii) Brazil [21], (iii) Ruhr [22] and (iv) Texas [23] containing a σ -term for $\pi - N$ scattering. They also consider the Fujita-Miyazawa force [24] dropping s-wave pions and Urbana-Argonne model [5] with additional isospin- and spin- independent components added to the Fujita-Miyazawa force.

In the Tucson-Melbourne (TM) model, the s-wave NNN force is written in the form

$$B(\mathbf{r}_{ij}, \mathbf{r}_{jk}) \{ \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k \} \{ (S_{ij} + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(S_{jk} + \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k) \}, \quad (12)$$

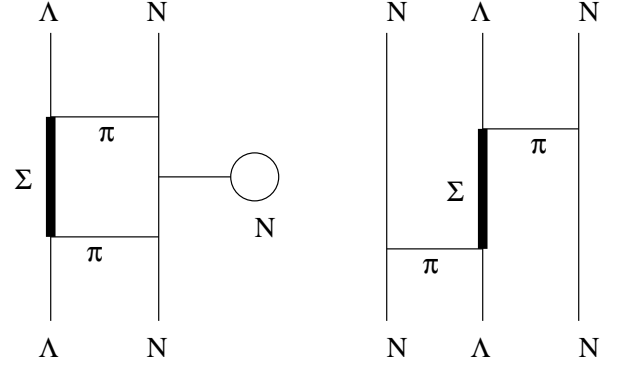


FIG. 1: Diagram representing $V_{\Lambda NN}^D$ and $V_{\Lambda NN}^{2\pi}$.

where, $B(\mathbf{r}_{ij}, \mathbf{r}_{jk})$ has several terms given in Ref. [5]. Recently, this has been expressed retaining only the term with pion-exchange-range functions as [25]

$$A^S = \left(\frac{f_{\pi NN}}{4\pi} \right)^2 a' m_\pi^2, \quad (13)$$

$$O_{ijk}^S = \sum_{cyc} Z(m_\pi r_{ij}) Z(m_\pi r_{jk}) \boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{kj} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k. \quad (14)$$

The parameter a' , whose value ranges from $-0.51/m_\pi$ to $-1.87/m_\pi$, is listed in Ref. [19]. The TM value $a' = -1.03/m_\pi$ gives the strength $|A^S| \approx 0.8 \text{ MeV}$. However, in many others it is assumed to have a value of 1.0 MeV .

Comparing TM model with the Eq. 6 for ΛNN potential, one may write an identical structure for both s-wave ΛNN and NNN potentials as following

$$C^S O_{\Lambda jk}^S \equiv A^S O_{ijk}^S. \quad (15)$$

This directly relates C^S in the strange sector to A^S in the non-strange sector. Since $\Lambda - N$ mass difference is small compared to the $\Delta - N$ mass difference, ΛNN potential of $S = -1$ sector is stronger than its non-strange counterpart NNN potential [18] of $S = 0$ sector. This provides stronger strengths in the case of ΛNN potential compared to the NNN potential. We, therefore, expect that the value of C^S would be more than 1.0 MeV , and is taken to be 1.5 MeV .

The charge symmetric ΛN potential [26, 27] reads as

$$v_{\Lambda N}(r) = v_0(r)(1 - \varepsilon + \varepsilon P_x) + (v_\sigma/4) T_\pi^2(m_\pi r) \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N. \quad (16)$$

The first term includes direct potential ($v_0(r) = v_c(r) - v_{2\pi}(r)$) and space-exchange potential ($\varepsilon v_0(r)(P_x - 1)$). Here, ε determines the odd-state potential, which is the strength of the space-exchange potential relative to the direct potential. Its estimate from the Λp forward-backward asymmetry is poor that ranges from 0.1 to 0.38 [16]. The potential $v_c(r) = W_c/[1 + \exp\{(r-R)/ar\}]$ is the Saxon-Woods repulsive potential, with $W_c = 2137 \text{ MeV}$, $R = 0.5 \text{ fm}$ and $a = 0.2 \text{ fm}$, and $v_{2\pi} = \bar{v} T_\pi^2(m_\pi r)$

TABLE I: ΛN potential strengths in units of MeV.

	v_s	v_t	$\bar{v} = (v_s + 3v_t)/4$	$v_\sigma = v_s - v_t$
$\bar{v}1$	6.33	6.09	6.15	0.24
$\bar{v}2$	6.28	6.04	6.10	0.24
$\bar{v}3$	6.23	5.99	6.05	0.24

is the two-pion attractive potential. The constants, $\bar{v} = (v_s + 3v_t)/4$ and $v_\sigma = v_s - v_t$, are respectively the spin-average and spin-dependent strengths, with $v_{s(t)}$ the singlet(triplet) state potential depth.

We perform variational Monte Carlo study to calculate the ground-state energy, $E = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle$, where Ψ is the WF of the hypernucleus. The computational details are available in Ref. [1]. For the spin-zero core nucleus, the expectation value of the spin part of the ΛN potential is negligibly small [1, 28, 29, 30]. So is the s-wave part of ΛNN potential. Moreover, correlations induced by them are too weak to offer any significant change in the energy. Thus, we choose a reasonable strength for these two. The energy is very sensitive to changes in the ΛN potential strengths \bar{v} and ε . They implicitly appear in the WF through the ΛN central and the *SEC* correlations. The ΛNN potential and its correlations involving C^P and W^D play an important role. Therefore, E or B_Λ are sensitive to the strengths \bar{v} , ε , C^P and W^D .

The value of $\bar{v} \approx 6.15(5)$ MeV is found consistent with the low energy Λp scattering data [17]. We use three different sets of v_s and v_t , which give three different values of \bar{v} and a constant v_σ as in Table I, referred to as $\bar{v}1$, $\bar{v}2$ and $\bar{v}3$. For all these, we choose three values of ε in the range from 0.1 to 0.38 as mentioned before. These are 0.1, 0.2 and 0.3. Results for these values are given in Tables II, III and IV.

The correlations induced by different components of ΛNN potential is written using scaled pair distances (\bar{r}) and a variational parameter δ^m as in Ref. [28],

$$U_{\Lambda ij} = \sum_m U_{\Lambda ij}^m = \sum_{m=D,P,S} \delta^m V^m(\bar{r}_{\Lambda i}, \bar{r}_{ij}, \bar{r}_{j\Lambda}). \quad (17)$$

The $\langle V_{\Lambda ij}^D \rangle$ obeys a linear behaviour, $\partial V_{\Lambda ij}^D / \partial W^D = \partial E / \partial W^D = \text{constant}$ at a fixed ε as it is not sensitive to the operators but only to *SEC* and hence to ε . As is obvious from Eq. 17, a change in the strength C^P offsets the WF. We observe that along with its own correlation parameter δ^P a couple of other parameters are found to change with C^P . But the repulsive ΛNN correlation, $U_{\Lambda NN}^D$, remains invariant. We perform calculations for a wide range of C^P starting from 0.5 MeV and upto 2.0 MeV. Therefore, enhancements in the attraction due to increasing C^P needs to be balanced by an appropriate increase in W^D . For every independent calculation, we tune the WF afresh and adjust the repulsive strength W^D in order to reproduce the experimental B_Λ . For the new $W^D(\text{new})$, we may easily obtain new $\delta^D(\text{new})$ using

$$\delta^D(\text{old})W^D(\text{old}) = \delta^D(\text{new})W^D(\text{new}), \quad (18)$$

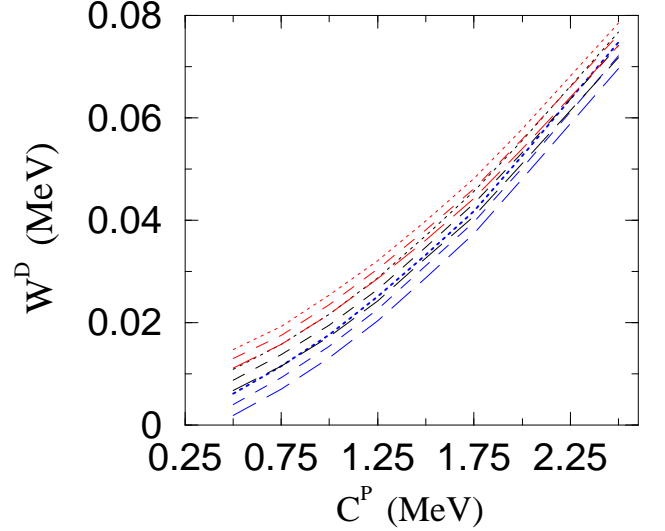


FIG. 2: Curves show the set of strengths giving B_Λ^{exp} . The dotted, dashed and long dashed lines represent $\varepsilon=0.1, 0.2$ and 0.3 , and the red, black and blue colors represent $\bar{v}1$, $\bar{v}2$ and $\bar{v}3$, respectively.

as $U_{\Lambda NN}^D$ is constant with C^P . Thus δ^D decreases in the same proportion W^D increases. The same is not true in the case of δ^P , which is found to be almost constant even if we multiply C^P by a factor of 4. This is because of the sensitivity of $V_{\Lambda ij}^P$ to its correlation, which is so strong that the attraction, $\langle V_{\Lambda ij}^{2\pi} \rangle$, increases more than 12 times for the corresponding 4 times increase in C^P . This quadratic behaviour is observed for all the ε and all the \bar{v} . The respective increase in the NC part of the energy (E_{NC}) is about 4 MeV. The T_Λ and $v_{\Lambda i}$ also exhibit considerable change due to the variation in C^P .

Solutions of all these strengths reproducing B_Λ^{exp} are plotted in Fig. 2. Thus, following the curves one may find potential strengths that reproduce B_Λ^{exp} . For every \bar{v} and at a fix value of C^P , we observe a linear relationship, $\partial W^D / \partial \varepsilon \approx c$, which is a consequence of two other linear relationships: (i) $\partial E / \partial \varepsilon = -\partial B_\Lambda / \partial \varepsilon \approx c_1$ (ii) and $\partial V_{\Lambda ij}^D / \partial W^D = -\partial E / \partial W^D = \partial B_\Lambda / \partial W^D \approx c_2$. The slope, $c = c_1 / c_2$ is found to increase with C^P , but only slightly (Table V). Curves representing different \bar{v} are found to get closer at higher C^P . Because, the dependence of energy on C^P as well as on W^D varies with \bar{v} . To match an increase in the attractive C^P we require a larger increase in the repulsive W^D for smaller \bar{v} .

This study has established the value of parameters appearing in the two- and three-body potentials in the strange sector. Furthermore, range of variations of these parameters is now, atleast roughly, known. In order to establish these parameters over a range of hypernuclei, numerous light and heavy hypernuclei have to be studied. This would help us to understand the variation of ΛN and ΛNN potentials as a function of density. Such a study would be in parallel to that of NN and NNN po-

tentials in nuclear systems where density variation plays an important role. A proper understanding of hypernuclei would help us to clarify the behaviour of the nuclear forces in the strange sector. Ultimately, a detailed study may lead us to a clarification of the role of QCD in determining the potential strengths. New results expected

from Japan Hadron Facility would help to sort out these questions in the near future.

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TABLE II: Energy breakdown for the set $\overline{v}1$ ($\overline{v} = 6.15$ and $v_\sigma = 0.24$). All quantities are in units of MeV except for ε .

	$\varepsilon = 0.1$				$\varepsilon = 0.2$				$\varepsilon = 0.3$			
	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$
T_Λ	8.56(3)	8.97(3)	9.83(3)	10.46(4)	8.23(3)	8.76(3)	9.38(3)	10.02(4)	7.94(3)	8.39(3)	9.01(3)	9.58(3)
$v_0(r)(1 - \varepsilon)$	-16.08(6)	-16.36(6)	-16.78(6)	-17.03(6)	-13.69(5)	-14.07(5)	-14.37(5)	-14.51(5)	-11.55(4)	-11.91(4)	-12.06(4)	-12.22(5)
$v_0(r)\varepsilon P_x$	-1.56(1)	-1.58(1)	-1.61(1)	-1.63(1)	-2.97(1)	-3.04(1)	-3.09(1)	-3.11(1)	-4.27(2)	-4.40(2)	-4.43(2)	-4.47(2)
$(\frac{1}{4})v_\sigma T_\pi^2(r)\sigma_\Lambda \cdot \sigma_i$	0.014(0)	0.015(0)	0.017(0)	0.017(0)	0.012(0)	0.013(0)	0.014(0)	0.014(0)	0.009(0)	0.010(0)	0.011(0)	0.011(0)
$v_{\Lambda i}$	-17.63(6)	-17.93(6)	-18.38(6)	-18.65(7)	-16.65(6)	-17.10(6)	-17.42(6)	-17.61(7)	-15.80(6)	-16.30(6)	-16.48(6)	-16.68(7)
$V_{\Lambda ij}^D$	2.49(1)	4.60(1)	7.99(4)	12.17(6)	2.16(1)	4.20(1)	7.35(4)	11.53(6)	1.80(1)	3.81(1)	6.81(4)	10.65(5)
$V_{\Lambda ij}^P$	-1.36(1)	-4.40(2)	-10.02(4)	-15.99(6)	-1.36(1)	-4.54(2)	-9.42(4)	-15.35(6)	-1.28(1)	-4.16(2)	-8.97(4)	-14.38(6)
$V_{\Lambda ij}^S$	-0.025(0)	0.017(1)	0.066(1)	0.100(1)	-0.025(0)	0.009(1)	0.057(1)	0.087(1)	-0.030(1)	0.009(1)	0.050(1)	0.077(1)
$V_{\Lambda ij}^{2\pi} = V_{\Lambda ij}^P + V_{\Lambda ij}^S$	-1.38(1)	-4.39(2)	-9.95(4)	-15.89(6)	-1.39(1)	-4.52(2)	-9.36(4)	-15.26(6)	-1.31(1)	-4.16(2)	-8.92(4)	-14.30(6)
$V_{\Lambda ij} = V_{\Lambda ij}^D + V_{\Lambda ij}^{2\pi}$	1.11(1)	0.21(2)	-1.96(2)	-3.73(3)	0.77(1)	-0.33(2)	-2.05(2)	-3.73(3)	0.49(1)	-0.34(2)	-2.11(2)	-3.65(3)
$V_\Lambda = v_{\Lambda i} + V_{\Lambda ij}$	-16.52(6)	-17.72(6)	-20.34(6)	-22.38(7)	-15.88(6)	-17.43(6)	-19.43(6)	-21.34(7)	-15.31(6)	-16.64(6)	-18.59(6)	-20.33(7)
$E_\Lambda = T_\Lambda + V_\Lambda$	-7.96(3)	-8.75(4)	-10.51(4)	-11.92(5)	-7.64(4)	-8.67(4)	-10.05(4)	-11.32(5)	-7.37(4)	-8.26(4)	-9.58(4)	-10.75(5)
T_{NC}	117.59(15)	118.42(15)	118.69(15)	119.26(15)	116.97(15)	117.84(15)	118.42(15)	118.84(15)	117.51(15)	118.43(15)	118.07(15)	118.53(15)
v_{NN}	-134.65(14)	-134.53(14)	-133.18(14)	-132.55(14)	-134.36(14)	-134.05(14)	-133.41(14)	-132.79(14)	-134.99(14)	-134.85(14)	-133.42(14)	-132.87(14)
V_{NNN}	-5.81(2)	-5.98(2)	-5.84(2)	-5.66(2)	-5.82(2)	-5.97(2)	-5.83(2)	-5.67(2)	-5.99(2)	-6.15(2)	-5.93(2)	-5.76(2)
$V_{NC} = v_{ij} + V_{ijk}$	-140.46(14)	-140.51(14)	-139.02(14)	-138.21(14)	-140.16(14)	-140.03(14)	-139.24(14)	-138.46(14)	-140.98(14)	-141.00(14)	-139.35(14)	-138.63(14)
$E_{NC} = T_{NC} + V_{NC}$	-22.88(4)	-22.10(5)	-20.32(4)	-18.95(5)	-23.21(4)	-22.18(5)	-20.82(4)	-19.52(5)	-23.47(4)	-22.58(5)	-21.27(4)	-20.11(5)
$E = E_\Lambda + E_{NC}$	-30.84(2)	-30.85(2)	-30.84(3)	-30.87(4)	-30.85(2)	-30.85(2)	-30.86(3)	-30.84(4)	-30.84(2)	-30.84(2)	-30.86(3)	-30.86(4)

TABLE III: Energy breakdown for the set $\overline{v}2$ ($\overline{v} = 6.10$ and $v_\sigma = 0.24$). All quantities are in units of MeV except for ε .

	$\varepsilon = 0.1$				$\varepsilon = 0.2$				$\varepsilon = 0.3$			
	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$
T_Λ	8.09(3)	8.54(3)	9.19(3)	9.91(3)	7.80(3)	8.15(3)	8.65(3)	9.51(3)	7.57(3)	7.94(3)	8.44(3)	9.18(3)
$v_0(r)(1 - \varepsilon)$	-14.61(5)	-14.79(5)	-15.13(5)	-15.48(5)	-12.54(5)	-12.64(5)	-12.94(5)	-13.30(5)	-10.59(4)	-10.73(4)	-11.04(5)	-11.15(4)
$v_0(r)\varepsilon P_x$	-1.41(1)	-1.42(1)	-1.45(1)	-1.47(1)	-2.70(1)	-2.72(1)	-2.77(1)	-2.83(1)	-3.88(2)	-3.93(2)	-4.03(2)	-4.03(2)
$(\frac{1}{4})v_\sigma T_\pi^2(r)\sigma_\Lambda \cdot \sigma_i$	0.007(0)	0.007(0)	0.008(0)	0.009(0)	0.007(0)	0.008(0)	0.008(0)	0.009(0)	0.005(0)	0.005(0)	0.006(0)	0.007(0)
$v_{\Lambda i}$	-16.02(6)	-16.20(6)	-16.57(6)	-16.94(6)	-15.23(6)	-15.35(6)	-15.70(6)	-16.12(6)	-14.46(6)	-14.65(6)	-15.04(6)	-15.18(6)
$V_{\Lambda ij}^D$	1.65(1)	3.46(2)	6.43(4)	10.39(5)	1.32(1)	3.09(2)	5.95(4)	9.81(5)	0.99(1)	2.71(2)	5.47(4)	9.22(5)
$V_{\Lambda ij}^P$	-1.33(1)	-4.36(2)	-9.00(4)	-14.95(6)	-1.23(1)	-4.04(2)	-8.08(3)	-14.26(6)	-1.19(1)	-3.98(2)	-7.61(3)	-13.84(6)
$V_{\Lambda ij}^S$	-0.028(0)	0.014(1)	0.049(1)	0.087(0)	-0.035(0)	0.007(0)	0.040(1)	0.078(0)	-0.032(0)	0.009(0)	0.036(1)	0.074(0)
$V_{\Lambda ij}^{2\pi} = V_{\Lambda ij}^P + V_{\Lambda ij}^S$	-1.35(1)	-4.35(2)	-8.59(4)	-14.86(6)	-1.27(1)	-4.04(2)	-8.04(4)	-14.18(6)	-1.22(1)	-3.97(2)	-7.80(4)	-13.77(6)
$V_{\Lambda ij} = V_{\Lambda ij}^D + V_{\Lambda ij}^{2\pi}$	-0.30(1)	-0.88(2)	-2.53(2)	-4.47(3)	0.05(1)	-0.94(2)	-2.09(2)	-4.37(3)	0.49(1)	-1.26(2)	-2.33(2)	-4.55(3)
$V_\Lambda = v_{\Lambda i} + V_{\Lambda ij}$	-15.72(6)	-17.08(6)	-19.10(6)	-21.41(6)	-15.18(6)	-16.29(6)	-17.79(6)	-20.49(6)	-14.69(6)	-15.91(6)	-17.37(6)	-19.73(6)
$E_\Lambda = T_\Lambda + V_\Lambda$	-7.62(3)	-8.54(4)	-9.90(4)	-11.51(5)	-7.37(3)	-8.14(4)	-9.15(4)	-10.98(5)	-7.13(4)	-7.97(4)	-8.93(4)	-10.54(5)
T_{NC}	116.82(15)	116.53(15)	117.36(15)	118.09(15)	116.73(15)	116.68(15)	117.19(15)	117.55(15)	116.81(15)	117.03(15)	117.16(15)	117.40(15)
v_{NN}	-134.22(14)	-132.90(14)	-132.59(14)	-131.80(14)	-134.40(14)	-133.50(14)	-133.12(14)	-131.82(14)	-134.43(14)	-133.72(14)	-133.12(14)	-131.96(14)
V_{NNN}	-5.83(2)	-5.92(2)	-5.74(2)	-5.62(2)	-5.81(2)	-5.90(2)	-5.78(2)	-5.60(2)	-6.10(2)	-6.21(2)	-5.94(2)	-5.73(2)
$V_{NC} = v_{ij} + V_{ijk}$	-140.05(14)	-138.82(15)	-139.31(15)	-137.42(14)	-140.21(14)	-139.41(14)	-138.90(15)	-137.42(14)	-140.53(14)	-139.93(15)	-133.12(15)	-137.69(14)
$E_{NC} = T_{NC} + V_{NC}$	-23.22(4)	-22.31(5)	-20.97(5)	-19.32(5)	-23.47(4)	-22.71(5)	-21.71(5)	-19.87(5)	-23.71(4)	-22.89(4)	-21.91(4)	-20.29(5)
$E = E_\Lambda + E_{NC}$	-30.84(2)	-30.85(2)	-30.87(2)	-30.84(3)	-30.85(2)	-30.85(2)	-30.86(2)	-30.85(3)	-30.84(2)	-30.87(2)	-30.84(2)	-30.84(3)

TABLE IV: Energy breakdown for the set $\bar{v}3$ ($\bar{v} = 6.05$ and $v_\sigma = 0.24$). All quantities are in units of MeV except for ε .

	$\varepsilon = 0.1$				$\varepsilon = 0.2$				$\varepsilon = 0.3$			
	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$	$C^p = .5$	$C^p = 1.0$	$C^p = 1.5$	$C^p = 2.0$
T_Λ	7.60(3)	8.09(3)	8.66(3)	9.18(3)	7.27(3)	7.73(3)	8.14(3)	8.87(3)	7.11(3)	7.45(3)	7.90(3)	8.48(3)
$v_0(r)(1 - \varepsilon)$	-13.26(5)	-13.57(5)	-13.69(5)	-13.95(5)	-11.20(4)	-11.48(5)	-11.64(5)	-12.02(5)	-9.58(4)	-9.68(4)	-9.90(4)	-9.94(4)
$v_0(r)\varepsilon P_x$	-1.27(1)	-1.30(1)	-1.30(1)	-1.31(1)	-2.40(1)	-2.45(1)	-2.48(1)	-2.54(1)	-3.49(2)	-3.51(2)	-3.56(2)	-3.58(2)
$(\frac{1}{4})v_\sigma T_\pi^2(r)\sigma_\Lambda \cdot \sigma_i$	-0.003(0)	-0.002(0)	-0.002(0)	-0.001(0)	0.003(0)	0.004(0)	0.005(0)	0.006(0)	0.002(0)	0.002(0)	0.003(0)	0.003(0)
$v_{\Lambda i}$	-14.53(6)	-14.87(6)	-15.00(6)	-15.27(6)	-13.60(6)	-13.92(6)	-14.10(6)	-14.56(6)	-13.06(6)	-13.19(6)	-13.48(6)	-13.52(6)
$V_{\Lambda ij}^D$	0.87(0)	2.61(1)	5.30(3)	8.79(5)	0.51(0)	2.16(1)	4.68(3)	8.25(5)	0.24(0)	1.82(1)	4.31(3)	7.50(4)
$V_{\Lambda ij}^P$	-1.20(1)	-4.07(2)	-8.53(4)	-13.45(6)	-1.11(1)	-3.84(2)	-7.49(4)	-12.87(6)	-1.06(1)	-3.62(2)	-7.24(4)	-12.27(6)
$V_{\Lambda ij}^S$	-0.032(0)	0.010(0)	0.046(0)	0.071(1)	-0.037(0)	0.003(0)	0.030(0)	0.062(1)	-0.037(0)	0.001(0)	0.029(0)	0.060(1)
$V_{\Lambda ij}^{2\pi} = V_{\Lambda ij}^P + V_{\Lambda ij}^S$	-1.23(1)	-4.06(2)	-6.48(4)	-13.37(6)	-1.15(1)	-3.84(2)	-7.46(4)	-12.81(6)	-1.09(1)	-3.63(2)	-7.21(4)	-12.21(6)
$V_{\Lambda ij} = V_{\Lambda ij}^D + V_{\Lambda ij}^{2\pi}$	-0.37(1)	-1.44(2)	-3.19(2)	-4.59(3)	-0.64(1)	-1.68(2)	-2.78(2)	-4.56(3)	-0.85(1)	-1.81(2)	-2.90(2)	-4.71(3)
$V_\Lambda = v_{\Lambda i} + V_{\Lambda ij}$	-14.89(7)	-16.32(6)	-18.19(6)	-19.86(6)	-14.24(7)	-15.60(6)	-16.89(6)	-19.12(6)	-13.91(7)	-11.48(6)	-16.38(6)	-18.23(6)
$E_\Lambda = T_\Lambda + V_\Lambda$	-7.29(3)	-8.23(3)	-9.52(4)	-10.67(4)	-6.97(3)	-7.87(4)	-8.75(4)	-10.25(4)	-6.80(3)	-7.55(4)	-8.48(4)	-9.74(4)
T_{NC}	115.97(15)	116.33(15)	116.54(15)	116.42(15)	115.28(15)	115.70(15)	116.01(15)	116.81(15)	116.04(15)	116.28(15)	116.23(15)	115.68(15)
v_{NN}	-133.81(14)	-133.07(14)	-132.06(14)	-131.98(14)	-133.44(14)	-132.81(14)	-132.32(14)	-131.73(14)	-134.04(14)	-133.41(14)	-132.63(14)	-131.08(14)
V_{NNN}	-5.73(2)	-5.88(2)	-5.80(2)	-5.61(2)	-5.73(2)	-5.87(2)	-5.75(2)	-5.67(2)	-6.03(2)	-6.16(2)	-5.95(2)	-5.71(2)
$V_{NC} = v_{ij} + V_{ijk}$	-139.53(14)	-138.95(14)	-137.86(14)	-136.59(14)	-139.17(14)	-138.68(14)	-138.07(14)	-137.40(14)	-140.08(14)	-139.57(14)	-138.58(14)	-136.79(14)
$E_{NC} = T_{NC} + V_{NC}$	-23.57(4)	-22.61(5)	-21.32(4)	-20.17(5)	-23.89(4)	-22.98(4)	-22.09(4)	-20.59(5)	-24.04(4)	-23.29(4)	-22.36(4)	-21.11(5)
$E = E_\Lambda + E_{NC}$	-30.85(2)	-30.84(2)	-30.84(2)	-30.84(3)	-30.85(2)	-30.85(2)	-30.84(2)	-30.84(3)	-30.84(2)	-30.84(2)	-30.85(2)	-30.86(3)

TABLE V: Variation of the slope, $\partial W^D/\partial\varepsilon$, with C^P and \overline{v} .

C^P (MeV)	$\overline{v}1$ (MeV)	$\overline{v}2$ (MeV)	$\overline{v}3$ (MeV)
0.5	-0.016(1)	-0.017(1)	-0.017(1)
1.0	-0.017(1)	-0.019(1)	-0.019(1)
1.5	-0.019(1)	-0.022(1)	-0.023(1)
2.0	-0.021(1)	-0.023(1)	-0.024(1)
2.5	-0.022(1)	-0.025(1)	-0.026(1)